**CSC 503 Homework Assignment 6**

Out: September 23, 2015

Due: September 30, 2015

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1. Open formula: P(x, y) v (Q(x, y) ᴧ ¬R(y, z))

Using Distributive law, above formula can be rewritten as

(P(x, y) v Q(x, y)) ᴧ (P(x, y) v ¬R(y, z))

* 1. Thus the clausal form using set notation is : {{P(x, y), Q(x, y)}, {P(x, y), ¬R(y, z)}}
  2. The above formula can be written so that variable can be standardized apart as :

{{P(x1, y1), Q(x1, y1)}, {P(x2, y2), ¬R(y2, z2)}}

1. Using the rules for negation normal form ∃x¬( P(x, y) ↔ ∀y Q(x, y) ) can be converted to NNF as per following steps:

**Step 1**: Ǝ x ¬ ((P(x, y) → ∀ y Q(x, y)) ᴧ (∀ y Q(x, y) → P(x, y)))

Since v ↔ w ⇒ (v → w) ᴧ (v → w)

**Step 2**: Ǝ x ¬ ((¬P(x, y) v ∀ y Q(x, y)) ᴧ (¬∀ y Q(x, y) v P(x, y)))

Since u→ w ⇒ ¬ u v w

**Step 3**: Ǝ x ¬ ((¬P(x, y) v ∀ y Q(x, y)) ᴧ (Ǝ y ¬Q(x, y) v P(x, y)))

Since Qx ¬ ∀y φ ⇒ Qx ∃y ¬φ NOTE: Qx stands for quantifier x.

**Step 4**: Ǝ x (¬ (¬P(x, y) v ∀ y Q(x, y)) v ¬ (Ǝ y ¬Q(x, y) v P(x, y)))

Since ¬ (u ᴧ w) ⇒ to ¬u v ¬w using De-Morgan’s law

**Step 5**: Ǝ x ((P(x, y) ᴧ ¬ ∀ y Q(x, y)) v (¬ Ǝ y ¬Q(x, y) v ¬P(x, y)))

Since using De Morgan’s Law: ¬ (u v w) is equivalent to ¬u ᴧ ¬w

**Step 6**: Ǝ x ((P(x, y) ᴧ Ǝ y ¬ Q(x, y)) v (∀ y Q(x, y) ᴧ ¬P(x, y)))

1. The formula ∀x((∃y P(x, y)) → Q(x, z)) ∧ ∃x((∀y R(x, y)) ∨ Q(x, y)) can be converted to prenex normal form as per following steps:

**Step 1**: ∀x (¬ (Ǝ y P(x, y)) v Q(x, z)) ᴧ Ǝ x ((∀y R(x, y)) v Q(x, y))

Since a → b ⇒ ¬ a v b

**Step 2**: ∀x ((∀y ¬ P(x, y)) v Q(x, z)) ᴧ Ǝ x ((∀y R(x, y)) v Q(x, y))

SinceQx ¬ ∃y φ ⇒ Qx ∀y ¬ φ

**Step 3**: ∀x ∀t ((¬ P(x, t/y)) v Q(x, z)) ᴧ Ǝ x ∀s (R(x, s/y) v Q(x, y))

SinceQx (∀y φ ∨ ψ) ⇒ Qx ∀z (φ(z/y) ∨ ψ)

**Step 4**: ∀x ∀t ∀s Ǝ w ((¬ P(x, t/y)) v Q(x, z)) ᴧ (R (w/x, s/y) v Q(w/x, y))

1. The formula ∀x ∃y P(x, y) → ∀x¬∀y Q(x, y) ∧ ¬∃x ∃y P(x, y) can be converted in Skolem Normal form as per following steps:

**Step 1**: ¬ (∀x Ǝ y P(x, y)) v ((∀x ¬∀y Q(x, y)) ᴧ (¬Ǝ x Ǝ y P(x, y)))

Since u→ w ⇒ ¬ u v w

**Step 2**: (Ǝ x ¬ Ǝ y P(x, y)) v ((∀x Ǝ y ¬Q(x, y)) ᴧ (∀x ¬Ǝ y P(x, y)))

Since Qx¬ ∀y φ ⇒ Qx ∃y ¬ φ and Qx ¬ ∃y φ ⇒ Qx ∀y ¬ φ

**Step 3**: (Ǝ x ∀y ¬P(x, y)) v ((∀x Ǝ y ¬Q(x, y)) ᴧ (∀x ∀y ¬P(x, y)))

Since Qx ¬ ∃y φ ⇒ Qx ∀y ¬ φ

**Step 4**: (∀y ¬P (f(y), y)) v ((∀x ¬Q(x, f(x))) ᴧ (∀x ∀y ¬P(x, y)))

1. θ = {f(y)/x, g(z)/y, v/w}

σ = {a/x, b/y, f(y)/z, w/v, c/u}

θσ **=** {f(b)/x, g(f(y))/y, f(y)/z, w/v, c/u}